

E-FIELD FROM AN ELECTRICALLY SHORT DIPOLE AS A FUNCTION OF VOLTAGE ON THE DIPOLE

(HANDOUT)

The θ component of the electric field from a small dipole ($L \ll \lambda$) with θ set to 90° (orthogonal direction to the dipole) is given by the following expression as a function of current on the dipole:

$$E_{\theta} = \frac{IL \sin \theta}{2\pi\epsilon} \left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3} \right)$$

Where:

I = Current in amperes

L = Length of dipole in meters

$\epsilon = 8.842 \times 10^{-12}$

θ = Angle to the plane of the dipole

r = Distance from the dipole in meters at which field is calculated

c = Speed of light in meters per second (3×10^8)

ω = Angular frequency of the signal exciting the dipole ($2\pi f$)

Since we are choosing the special case where $\theta = 90^\circ$ this equation becomes:

$$1.) E_{\theta} = \frac{IL}{2\pi\epsilon} \left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3} \right)$$

Rewriting (1.) for the purpose of separating it into its real and imaginary parts yields:

$$E_{\theta} = \frac{jIL}{2\pi\epsilon} \left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3} \right) + \frac{IL}{2\pi\epsilon} \left(\frac{1}{cr^2} \right)$$

or:

$$E_{\theta} = \frac{IL}{2\pi\epsilon} \left(j \left[\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3} \right) \right] + \frac{1}{cr^2} \right)$$

Therefore:

$$2.) |E_{\theta}| = \frac{IL}{2\pi\epsilon} \sqrt{\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3} \right)^2 + \left(\frac{1}{cr^2} \right)^2}$$

To get "I" in terms of volts impressed on the antenna, we need impedance "Z" of the antenna. This impedance has two components, a resistive (R_o) and an imaginary (X_o) component.

Therefore, the magnitude of the impedance "Z" may be expressed as:

$$3.) |Z| = \sqrt{R_o^2 + X_o^2}$$

And:

$$4.) R_o = 20 [\beta L]^2 \text{ and } \beta = \frac{2\pi}{\lambda}$$

$$5.) X_o = \frac{30}{\beta L} \left[-4 + 4 \ln \left(\frac{L}{A} \right) \right]$$

Where:

L= Length of dipole in meters

A= Diameter of the wire in meters

$$\lambda = 3 \times 10^8 / f_{\text{Hz}}$$

Substituting for β , (4.) becomes:

$$4.) R_o = 20 \left[\frac{2\pi}{\lambda} L \right]^2 = \frac{80\pi^2 L^2}{\lambda^2}$$

It is now possible to substitute for R_o and X_o in (3.) which yields:

$$6.) |Z| = \sqrt{\left(\frac{80\pi^2 L^2}{\lambda^2} \right)^2 + \left(\frac{15\lambda}{\pi L} \left[-4 + 4 \ln\left(\frac{L}{A} \right) \right] \right)^2}$$

Substituting V/Z for I in equation (2.) yields:

$$7.) |E_o| = \frac{V L}{2\pi\epsilon |Z|} \sqrt{\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3} \right)^2 + \left(\frac{1}{c r^2} \right)^2}$$

Where V = Dipole volts RMS

Other observations:

For electrically short dipoles, the R_o term in the antenna impedance equation is usually several orders of magnitude less than the value of X_o .

If $r \ll \lambda$ and $\omega \ll c^2$ then (7.) becomes:

$$8.) E_{\theta} = \frac{V L}{2 \pi \epsilon |Z| \omega r^3}$$

$$\text{And, } |Z| = X_o$$

This simplified equation (8.) was used in the calculations presented for electric field emissions.

Example:

1 Volt RMS, 1 MHz on a dipole 0.1 meter long X 1 millimeter in diameter measured at a distance 1 meter from the antenna = 1.38 millivolts/meter field strength (63 dB μ V/M)